SAMPLING FOR WEEKLY TIME SERIES

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Introduction

The problem we wish to discuss in this paper is the design of a general purpose repetitive sample survey to provide estimates of a number of statistics. The estimates from the survey on successive occasions are to be used to draw inferences about the population and underlying changes in the population. For this purpose they will be subject to a wide variety of analyses and comparisons, ranging from more or less formal time series analysis to comparisons with preceding weeks, preceding months, the preceding year, and patterns from earlier years. From time to time the data from a number of occasions may be pooled to provide estimates of aggregates or more detailed analyses of the characteristics and changes in characteristics of the population.

There are two interrelated aspects to the problem of survey design in these circumstances. One of these, to which we will give major attention, is the design of a sampling and estimation procedure for each occasion. The various demands on the survey lead to conflicting objectives from a sampling point of view, and the design to be chosen will generally represent a compromise. The other aspect of the problem of survey design is the frequency with which data are to be collected and estimates produced. Given fixed total resources, the more frequent the occasions at which data are to be collected the smaller the sample it is feasible to cover at each occasion. Thus, the question arises as to whether smaller samples should be taken more frequently or larger samples less frequently. This question needs more exploration by survey analysts.[1]

We will be especially interested in exploring the circumstances under which it may be advantageous to use a system of weekly samples even though the expressed primary interest of the survey lies in estimating monthly statistics. The techniques we will discuss are the use of rotating samples and composite estimates [1] [2] [3] [4]. The flexibility of rotating samples provides the survey designer with the opportunity to choose from a number of different plans to meet special circumstances and important demands for data. Among these choices is the opportunity to sample the occasions themselves. We will compare designs based on weekly estimates every week with a design based on a systematic sample of weeks. In the latter case there is a component of variation between weeks that is generally neglected but should be taken into account.

In comparing alternative sample designs we will take as our criterion the sampling variance of estimates from the survey. These estimates will be used in a wide variety of analyses, and variance appears to be one of the best general criteria for all uses.

Rotating Samples and Composite Estimation

Given a repetitive survey, at one extreme the sample at each occasion might be identical. This would be the case of a fixed panel. At the other extreme, the sample at each occasion might be completely independent. This would be the case of complete rotation of the sample. Cases in between represent partial or incomplete rotation of the sample. The pattern of occasions and number of occasions for which any given sampling unit provides data in the survey is the rotation plan. The significance of the rotation plan is that it determines the carryover of information between occasions by identical units.

It is helpful to distinguish the concept of overlap in information between occasions from sample overlap, that is, the extent to which identical sampling units are surveyed at different occasions. If information is obtained from sample units for only one occasion in the interview or report, the overlap in information between occasions is the same as the sample overlap. Overlap in information between different occasions can, however, be achieved without overlap in the sample interviewed if information for more than one occasion can be obtained from sample units in a single interview or report.

To illustrate these concepts, suppose a continuing interview survey based on equal-sized weekly samples of segments with the following rotation plan: Any one segment is in sample only one week of the year and then returns to the sample again in the corresponding week of the following year. At interview, information is obtained from each unit in the segment for each of the two weeks preceding the week of interview. Then the overlap in information between successive weeks is 50 percent and the overlap between corresponding weeks a year apart is 100 percent. This plan is discussed further below.

If we have overlap of information between occasions we are in a position to use data from the past to try to improve current estimates. Suppose we have a continuing weekly survey with overlap in information between successive weeks and we are interested in estimating the level of an item for the current week. We can construct an estimate of level in at least two different ways:

1. By taking the (composite) estimate of level for the preceding week and adding an estimate of change derived from the sampling units for which there is information for both weeks.
2. By making an estimate of level for the given week directly, using the data from all sampling units for which there is information for the current week.

The composite estimate of level for any given week is a weighted average of these two types of estimates. The composite estimate of change between any two weeks is the difference between the composite estimates of level for the two weeks.

The determination of the appropriate weights to use in the composite estimate for a single item is a straight forward problem in optimization. The appropriate weighting to minimize the sampling variance of the composite estimate will however vary from item to item, and between statistics of level and change for a given item, depending upon the pattern of correlations. The determination of an efficient rotation plan is also a problem in optimization. It is not, however, straight forward but has thus far been dealt with only by comparison of specific alternatives. If information for a sampling unit is positively correlated between different time periods, as one might expect a high degree of overlap is desirable for estimating change between time periods. This is not so for estimates of level, or for estimates of aggregates based on the sum of successive sample estimates or analyses for which successive samples are to be pooled. If overlap in information can be achieved in the interview by collecting data for more than one time period rather than by sample overlap, the objectives of estimating both level and change well can be more successfully reconciled than if we must depend on overlapping the sample.

Reasons for Using Rotating Samples with Composite Estimates

It is worthwhile to review in some detail the reasons for using rotating samples with composite estimates compared with the alternatives of a fixed panel or completely independent samples. These will be considered in connection with sampling variances, and measurement and control of response error.

With regard to sampling variances, it will generally be found that where partial rotation is of advantage over a fixed panel, say, a design based on completely independent samples has the same advantage to an even greater extent -- and a similar statement could be made in the opposite direction. Thus, we may think of the use of rotating samples with composite estimation as having some advantages and some disadvantages compared with alternative designs. It combines the possibilities available under either of the two extremes and, although it does not exploit any of them to the hilt, it is fortunate that frequently most of the benefit of a particular feature will be gained even without complete exploitation.

There are certain unique advantages in the measurement of response error that are inherent in the use of rotating samples compared with either a complete overlap of information or no overlap of information.

Sampling Variances

Compared with a fixed panel design rotating samples provide improved estimates of current level. This is so because the composite estimate makes it possible to take advantage of the information in past samples. This raises a further interesting possibility. Ordinarily, a time series is produced point by point as each point in time is reached. Suppose we are willing to revise the current estimates at a later date, say annually, with the object of developing the "best" historical series. The composite estimate can be extended so that the "current" estimate at each point in time takes advantage of the information in samples future to that time as well as in the past samples.

There is a further advantage with rotating samples in the ability to treat more satisfactorily unexpected large units that occur in the sample. When unexpectedly large observations occur in a sample survey a choice must ordinarily be made between accepting the considerable increase in variance they create or of reducing their weight and accepting the resulting bias. The usual advice is to choose the alternative expected to lead to a smaller mean square error. With rotating samples it is possible to improve on this procedure by identifying all large observations in the entire annual sample and including them in the survey for the current time period. The effect of this is to sample large observations at each occasion at a rate k times that of other observations, where k is the ratio of the number of different segments annually to the number in the current survey, and hence to divide their weight in the current estimate by k. While the mechanics of putting this principle into effect may sometimes require considerable ingenuity, the resulting gains can be substantial.

Measurement and Control of Response Errors

We will consider the problems of response error in connection with nonresponse, quality of data for interview cases and measurement of response differences.

Compared with a fixed panel design, the use of rotating samples reduces the burden of reporting on the individual respondent. This can be important in maintaining a high rate of response. Where there is nonresponse, rotating and completely dependent samples both have the advantage over completely independent samples in that an earlier report may be available to permit better imputation or adjustment for the nonresponse cases in the current estimate. This can be of special help for statistics of change between different points in time, since the impact of nonresponse on statistics of change may be more nearly measured by the sum of the nonresponse rates at each of the two occasions than by the individual rates.

Where a response is obtained, previous information may be useful for improving the

current information. This may be realized in any of several ways -- for example, by the use of shuttle forms where the respondent has the opportunity to see his earlier information, or by the application of editing rules leading to follow-up to correct or clarify the information originally reported.

Interview on successive occasions may make it possible to obtain better current information through improved techniques. For example, in pilot studies of the reporting of homeowners' expenditures for alterations and repairs it was found that more precise and complete reporting of smaller expenditures could be obtained by furnishing the interviewer with a copy of the previous response to read to the respondent and asking about expenditures since the previous interview.[6] This has been called a "bounded interview" since the earlier interview bounds the current one.

The opportunity to compare responses for identical units at different points in time with responses for new units that is provided by rotating samples may help to uncover defects in the survey procedure. This was the case in the Census Bureau's Monthly Retail Trade Survey. Data are obtained at interview in that survey for each of the two calendar months preceding the month of interview. It was found that the survey procedure tended to miss stores that had gone out of business during the month preceding interview. This was remedied in part by providing for special field instructions in case of vacant stores.

Where overlap of data is created by asking for information covering more than one time period, the comparison of data for a fixed period obtained from successive panels may point to problems of recall and suggest revisions in the survey procedure. In several household health surveys a recall period of four weeks was used for obtaining reports, with a system of independent weekly samples. Comparison of the data for each calendar week when it was the week preceding the week of interview, two weeks preceding the week of interview, etc., indicated a sharp decline in the level of illness reported with increasing length of recall. In the pilot study for the National Health Survey a two-week recall period was tested by this technique and found to be acceptable.[7]

There is a special aspect to surveying the same unit on more than one occasion, and that is the training or conditioning effect on the respondent. In the case of well-defined item, the effect of repeated interview and questioning of the respondent may reasonably be expected to lead to improved data. When the definition of the item has subjective elements, however, it may be questionable whether the data from later reports are better or worse. Examples can be cited on both sides of the argument. There is, however, ample experience to show that they will frequently be different. With rotating samples there is an opportunity to compare responses of new units in the survey with those of continuing units so that such differences can be measured and explored.

Some Choices among Alternative Sample Designs

We now turn to the specific question of the use of weekly samples posed at the beginning of this paper, and will examine some alternative survey designs (including different rotation plans) from a variance point of view. We will consider various statistics under a composite estimation procedure for two types of items.

We assume the following special conditions for the survey --

(1st) That respondents have satisfactory recall for the last two time periods (weeks) but cannot furnish satisfactory data for a longer period.

(2nd) That primary interest of the survey lies in estimating either the total value of an item for four time periods (month) or the average value of the item for four time periods and in the changes over time in these totals and averages.

(3rd) That the estimate of total or average for the item is to be published at the end of every four time periods on a timely basis. Changes in level are to be derived from the published totals or averages.

Many rotation patterns are available for use under these circumstances. Four possible plans of rotation will be described here. In three plans a predesignated number of interviews are made every week. These plans have a weekly overlap of 50 percent in information, since they involve asking the respondent for data in the last two weeks, but they differ from one another in either their monthly overlap or yearly overlap. In the fourth plan, a systematic sample of weeks is taken, one week from each month; four times the predesignated number of interviews are concentrated in the sample week and the respondent is asked for data for only the last week.

Description of the Rotation Plans

The first plan considered is the "50-75-50 Plan." This plan is characterized by a 50 percent overlap in information from week to week; a 75 percent overlap in sample from month to month, and a 50 percent overlap in sample from year to year. A respondent on his first interview furnishes separate data for the last two weeks; he is interviewed three more times, the interviews being spaced by four-week intervals, before he has an eight month respite. After this he is interviewed four more times at monthly intervals, so that the total number of interviews with this respondent is eight.

The interview pattern is sketched in Diagram I, where each line represents a different week and each column represents a different sample person (or group of persons). Different sample persons are designated in the diagram by different letters. As an illustration, the sample person or sample group designated by the small letter "m" is interviewed in weeks t, t-4, t-8, t-12, t-52, t-56, t-60, and t-64.

The second plan is the "50-0-100 Plan," indicating a 50 percent weekly overlap, 0 percent monthly overlap, and 100 percent yearly overlap. Each sample element is interviewed just one a year, in every year.

The third plan is the "50-50-100 Plan," and indicates 50 percent weekly overlap, 50 percent monthly overlap, and 100 percent yearly overlap. After the first interview, a sample respondent is interviewed four weeks later, is out of the sample for 10 months, then returns for two interviews, is out again for 10, etc. Each sample element is interviewed twice a year.

The fourth plan is the "X-75-50 Plan," under which interviews are conducted only in a systematic sample of weeks, one from each month. On interview, the sample person reports for the last week only and not for the last two weeks. The sample person is interviewed in the sample week in four consecutive months; he is out for eight months and then in again for four months. Thus, eight interviews are conducted for each sample element. As mentioned previously, the number of interviews in the sample week under this plan is equal to the number of interviews conducted in a month for one of the previous three rotation plans.

Results

The tables below compare the variances of estimates of different statistics for two different items as derived from estimating the weekly level of an item by means of the "composite" estimator employing a weight of 0.5. In the first three plans advantage is taken of those responses for the week being estimated obtained from the sample elements during interviews made in the subsequent week. Except for a factor of 1/n for sample size, the number 4.00 represents the variance of a simple unbiased estimate of weekly level of an item from an independent weekly sample of size n; or equivalently the number 1.00 represents the variance of the average of four such weekly levels during a month.

The following conclusions can be drawn from the tables:

(1) In general, the different plans have different abilities to estimate the different statistics. For level those plans are best which utilize more different and distinct respondents. In alternate terms those plans are best for level which involve fewest repetitions of interviews with the sampled elements.

(2) In general, for change between two time periods, those plans are best which have the highest overlap between the given time periods. This is somewhat conditioned by the fact that the plan must not lose too much on level because of repetitive interviews.

(3) That there may be purposes for which individual weekly levels themselves and changes in these levels have satisfactory variances even if the objective at the outset might have been to provide monthly averages or totals of the weekly levels. (This conclusion follows from the fact that the weekly levels generally have less than twice the variance of the monthly average.)

(4) Dependent on the size of the between week variance contribution, (and the size of sample employed) the "X-75-50 Plan" may result in poorer estimates of monthly averages or totals than any of the other three plans which do not involve sampling of weeks.

Footnote

[1] For a discussion of this question in the context of statistical quality control see [5].

References

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[3] Woodruff, R. S., "The Use of Rotating Samples in the Census Bureau's Monthly Surveys," Proc. Soc. Stat. Sec., Amer. Stat. Assn., 1959, 130-138.

[4] Hansen, M. H., Hurwitz, W. N., Nisselson, H. and Steinberg, J. "The redesign of the Census Current Population Survey," Journ. Amer. Stat. Assn., 50 (1955), 701-719.

[5] Weiler, H., "The Use of Runs to Control the Mean in Quality Control," Journ. Amer. Stat. Assn., 48 (1953), 439-446 (and Errata, 51 (1956) 652).

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[7] Nisselson, H. and Woolsey, T. D., "Some Problems of the Household Interview Design for the National Health Survey," Journ. Amer. Stat. Assn., 54 (1959), 69-87.

Table I

COMPARATIVE VARIANCES WITH FOUR DIFFERENT ROTATION PLANS FOR ESTIMATES OF LEVEL AND CHANGES OVER VARIOUS TIME PERIODS: A HIGH CORRELATION[a] ITEM

Statistic	Estimate of statistic based on -						
	Average of 4 weekly estimates			Individual weekly estimate(s)			
	50-75-50 Plan	50-0-100 Plan	50-50-100 Plan	50-75-50 Plan	50-0-100 Plan	50-50-100 Plan	X-75-50 Plan
Level.....	0.92	0.70	0.85	1.27	1.14	1.23	0.83[b]
Difference between two levels:							
1 month apart.....	0.59	1.05	0.82	1.08	2.09	1.46	0.55
1 year apart.....	1.23	0.49	0.57	1.75	0.88	0.94	1.14
1 week apart.....	c/	c/	c/	0.66	0.73	0.69	d/
3 months apart....	1.51	1.40	1.68	e/	e/	e/	1.22
6 months apart....	1.85	1.40	1.69	e/	e/	e/	1.58

a/ A high correlation item is one for which $\rho_1 = .95$, $\rho_3 = .85$, $\rho_4 = .80$, $\rho_5 = .75$, $\rho_7 = \rho_8 = \rho_9 = .70$, $\rho_{11} = \rho_{12} = \rho_{13} = .65$, $\rho_{51} = \rho_{52} = \rho_{53} = .70$, $\rho_{47} = \rho_{48} = \rho_{49} = \rho_{55} = \rho_{56} = \rho_{57} = .65$, $\rho_{43} = \rho_{44} = \rho_{45} = \rho_{59} = \rho_{60} = \rho_{61} = .60$, $\rho_{39} = \rho_{40} = \rho_{41} = \rho_{63} = \rho_{64} = \rho_{65} = .60$; where ρ_t is the correlation over a t week period.

b/ The estimate of weekly level can be made for but one week during the month. The estimate for the week can be used as the estimate for the average of the 4 weeks from which the week was sampled, in which case the variances in this column must be increased to reflect the between-week variance.

c/ Inapplicable

d/ Not possible

e/ Not available

Table II

COMPARATIVE VARIANCES WITH FOUR DIFFERENT ROTATION PLANS FOR ESTIMATES OF LEVEL AND CHANGES OVER VARIOUS TIME PERIODS: A LOW CORRELATION[a] ITEM

Statistic	Estimate of statistic based on -						
	Average of 4 weekly estimates			Individual weekly estimate(s)			
	50-75-50 Plan	50-0-100 Plan	50-50-100 Plan	50-75-50 Plan	50-0-100 Plan	50-50-100 Plan	X-75-50 Plan
Level.....	1.14	0.79	1.09	1.98	1.69	1.95	1.00[b]
Difference between two levels:							
1 month apart.....	1.28	1.27	1.41	2.93	3.21	3.16	1.16
1 year apart.....	1.83	0.91	1.37	3.40	2.23	2.82	1.62
1 week apart.....	[c]	[c]	[c]	1.81	2.02	1.82	[d]
3 months apart.....	2.10	1.57	2.16	[e]	[e]	[e]	1.73
6 months apart.....	2.27	1.57	2.17	[e]	[e]	[e]	1.96

a/ A low correlation item is one for which $\rho_1 = .70$, $\rho_3 = .56$, $\rho_4 = .50$, $\rho_5 = .44$, $\rho_7 = \rho_8 = \rho_9 = .40$, $\rho_{11} = \rho_{12} = \rho_{13} = .30$, $\rho_{51} = \rho_{52} = \rho_{53} = .50$, $\rho_{47} = \rho_{48} = \rho_{49} = \rho_{55} = \rho_{56} = \rho_{57} = .45$, $\rho_{43} = \rho_{44} = \rho_{45} = \rho_{59} = \rho_{60} = \rho_{61} = .40$, $\rho_{39} = \rho_{45} = \rho_{41} = \rho_{63} = \rho_{64} = \rho_{65} = .30$; where ρ_t is the correlation over a t week period.

b/ The estimate of weekly level can be made for but one week during the month. The estimate for the week can be used as the estimate for the average of 4 weeks from which the week was sampled, in which case the variances in this column must be increased to reflect the between-week variance.

c/ Inapplicable

d/ Not possible

e/ Not available

Diagram I -- R O T A T I O N P A T T E R N F O R T H E "50-75-50 PLAN"